Risk Aggregation with Dependence Uncertainty

Carole Bernard

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Risk Aggregation and Diversification

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- Using the standard deviation to measure the risk of aggregating X_1 and X_2 with standard deviation $std(X_i)$,

$$std(X_1 + X_2) = \sqrt{std(X_1)^2 + std(X_2)^2 + 2\rho std(X_1)std(X_2)}$$

If $\rho < 1$, there are "diversification benefits":

$$std(X_1 + X_2) < std(X_1) + std(X_2)$$

Risk Aggregation and Diversification

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$$std(X_1 + X_2) = \sqrt{std(X_1)^2 + std(X_2)^2 + 2\rho std(X_1) std(X_2)}$$

If $\rho < 1$, there are "diversification benefits":

$$std(X_1 + X_2) < std(X_1) + std(X_2)$$

- This is not the case for instance for Value-at-Risk (but used in regulatory capital requirements).

Motivation on VaR aggregation with dependence uncertainty

Full information on **marginal distributions**:

$$X_j \sim F_j$$

$$+$$

Full Information on **dependence**:

(known copula)

$$\Rightarrow$$

$\mathrm{VaR}_q\left(X_1 + X_2 + ... + X_d\right)$ can be computed!

Motivation on VaR aggregation with dependence uncertainty

Full information on **marginal distributions**:

$$X_j \sim F_j$$

$$+$$

Partial or **no** Information on **dependence**:

(incomplete information on copula)

$$\Rightarrow$$

$\mathrm{VaR}_q\left(X_1 + X_2 + ... + X_d\right)$ **cannot** be computed!

Only a range of possible values for $\mathrm{VaR}_q\left(X_1 + X_2 + ... + X_d\right)$.

Objectives and Findings

- *Model uncertainty on the risk assessment* of an aggregate portfolio: the sum of d dependent risks.
    - Given all information available in the market, what can we say about the maximum and minimum possible values of a given risk measure of a portfolio?

Objectives and Findings

- *Model uncertainty on the risk assessment* of an aggregate portfolio: the sum of d dependent risks.
  - Given all information available in the market, what can we say about the maximum and minimum possible values of a given risk measure of a portfolio?
- Implications:
  - Current VaR based regulation is subject to high model risk, even
    - if one knows the multivariate distribution "almost completely" or
    - if one knows average pairwise correlation.

Acknowledgement of Collaboration (1/2)

with M. Denuit (UCL), X. Jiang (UW), L. Rüschendorf (Freiburg), S. Vanduffel (VUB), J. Yao (VUB), R. Wang (UW):

- Bernard, C., X. Jiang, R. Wang, (2013) *Risk Aggregation with Dependence Uncertainty*, **Insurance: Mathematics and Economics**.
- **Bernard, C., Vanduffel, S. (2015). A new approach to assessing model risk in high dimensions. Journal of Banking and Finance. (Part I)**
- Bernard, C. , Rüschendorf, L., Vanduffel, S., Yao, J. (2015). *How robust is the Value-at-Risk of credit risk portfolios?* **European Journal of Finance**.
- Bernard, C., Rüschendorf, L., Vanduffel, S. (2017). *Value-at-Risk bounds with variance constraints*. **Journal of Risk and Insurance**.
- Bernard, C., L. Rüschendorf, S. Vanduffel, R. Wang (2017) *Risk bounds for factor models,* 2017, **Finance and Stochastics**.
- Bernard, C., Denuit, M., Vanduffel, S. (2018). *Measuring Portfolio Risk Under Partial Dependence Information*. **Journal of Risk and Insurance**.

Acknowledgement of Collaboration (2/2)

More recently with two of my current PhD students, Corrado De Vecchi (VUB) and Rodrigue Kazzi (VUB):

- Bernard, C., R. Kazzi, S. Vanduffel (2020) *Range Value-at-Risk Bounds for Unimodal Distributions under Partial Information*, **Insurance: Mathematics and Economics**.
- Bernard, C., R. Kazzi, S. Vanduffel (2021) *A Practical Approach to Quantitative Model Risk Assessment*, **Forthcoming in Variance**.
- Bernard, C., R. Kazzi, S. Vanduffel (2021) *Model Uncertainty Assessment for Unimodal Symmetric and Log-Symmetric Distributions*, **Working Paper**.
- Bernard, C., C. De Vecchi, S. Vanduffel (2021) *How does correlation impact Value-at-Risk bounds*, **Working Paper to be Presented in Part III.**

Model Risk

1. Goal: Assess the risk of a portfolio sum $S = \sum_{i=1}^{d} X_i$.
2. Choose a risk measure $\rho(\cdot)$: variance, Value-at-Risk...
3. "Fit" a multivariate distribution for $(X_1, X_2, ..., X_d)$ and compute $\rho(S)$
4. How about model risk? How wrong can we be?

Model Risk

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4. How about model risk? How wrong can we be?

Assume $\rho(S) = var(S)$,

$$\rho^+_{\mathcal{F}} := \sup\left\{var\left(\sum_{i=1}^{d} X_i\right)\right\}, \quad \rho^-_{\mathcal{F}} := \inf\left\{var\left(\sum_{i=1}^{d} X_i\right)\right\}$$

where the bounds are taken over all other (joint distributions of) random vectors $(X_1, X_2, ..., X_d)$ that "agree" with the available information $\mathcal{F}$

Aggregation with dependence uncertainty: Example - Credit Risk

- **Marginals known**
- **Dependence fully unknown**

Consider a portfolio of 10,000 loans all having a default probability $p = 0.049$.

		Min VaR_q	Max VaR_q
q = 0.95		0%	98%
q = 0.995		4.4%	100%

Portfolio models are subject to significant model uncertainty (defaults are rare and correlated events).

Aggregation with dependence uncertainty: Example - Credit Risk

- **Marginals known**
- **Dependence fully unknown**

Consider a portfolio of 10,000 loans all having a default probability $p = 0.049$. *The default correlation is* $\rho = 0.0157$ *(for KMV).*

	KMV VaR_q	Min VaR_q	Max VaR_q
q = 0.95	10.1%	0%	98%
q = 0.995	15.1%	4.4%	100%

Portfolio models are subject to significant model uncertainty (defaults are rare and correlated events).
Using dependence information is crucial to try to get more "reasonable" bounds.

Outline of the Talk

Part 1: Bounds on Variance

- With full dependence uncertainty
- With partial dependence information on a subset

Part 2: Bounds on Value-at-Risk

- With 2 risks and full dependence uncertainty
- With d risks and full dependence uncertainty
- With partial dependence information on a subset

Part 3: Bounds on Value-at-Risk

- With 2 risks and information on pairwise correlation
- With d risks and information on average correlation

Part I

Bounds on variance

Risk Aggregation and full dependence uncertainty

- Marginals known:
- Dependence fully unknown
- In two dimensions $d = 2$, assessing model risk on variance is linked to the Fréchet-Hoeffding bounds

$$var(F_1^{-1}(U) + F_2^{-1}(1-U)) \leq var(X_1 + X_2) \leq var(F_1^{-1}(U) + F_2^{-1}(U))$$

- Maximum variance is obtained for the comonotonic scenario:

$$var(X_1 + X_2 + ... + X_d) \leq var(F_1^{-1}(U) + F_2^{-1}(U) + ... + F_d^{-1}(U))$$

- Minimum variance: A challenging problem in $d \geq 3$ dimensions
  - Wang and Wang (2011, JMVA): concept of complete mixability
  - Puccetti and Rüschendorf (2012): algorithm (RA) useful to approximate the minimum variance.

Bounds on variance

Analytical Bounds on Standard Deviation

Consider d risks X_i with standard deviation σ_i

$$0 \leq std(X_1 + X_2 + ... + X_d) \leq \sigma_1 + \sigma_2 + ... + \sigma_d.$$

Example with 20 normal N(0,1)

$$0 \leq std(X_1 + X_2 + ... + X_{20}) \leq 20,$$

in this case, both bounds are sharp and too wide for practical use!
THUS: Incorporate information on dependence.

Illustration with 2 risks with marginals N(0,1)

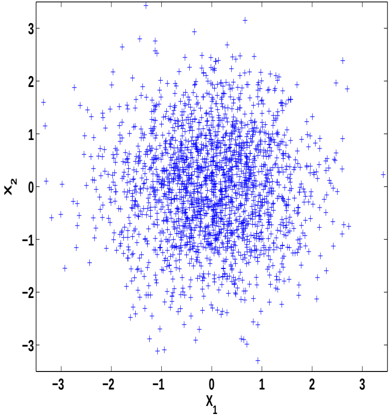
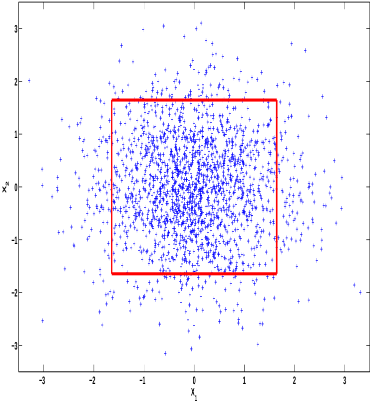


Illustration with 2 risks with marginals N(0,1)



Assumption: Independence on $\mathcal{F} = \bigcap_{k=1}^{2} \{q_\beta \leq X_k \leq q_{1-\beta}\}$.

Our assumptions on the cdf of $(X_1, X_2, ..., X_d)$

$\mathcal{F} \subset \mathbb{R}^d$ ("trusted" or "fixed" area)
$\mathcal{U} = \mathbb{R}^d \backslash \mathcal{F}$ ("untrusted").
We assume that we know:

(i) the marginal distribution F_i of X_i on $\mathbb{R}$ for $i = 1, 2, ..., d$,
(ii) the distribution of $(X_1, X_2, ..., X_d) \mid \{(X_1, X_2, ..., X_d) \in \mathcal{F}\}$.
(iii) $p_f := P\left((X_1, X_2, ..., X_d) \in \mathcal{F}\right)$.

- When only marginals are known: $\mathcal{U} = \mathbb{R}^d$ and $\mathcal{F} = \emptyset$.
- **Our Goal: Find bounds on** $\rho(S) := \rho(X_1 + ... + X_d)$ **when** $(X_1, ..., X_d)$ **satisfy (i), (ii) and (iii).**

Example $d = 20$ risks N(0,1)

- $(X_1, ..., X_{20})$ independent N(0,1) on

$$\mathcal{F} := [q_\beta, q_{1-\beta}]^d \subset \mathbb{R}^d \qquad p_f = P\left((X_1, ..., X_{20}) \in \mathcal{F}\right)$$

(for some $\beta \leq 50\%$) where q_γ: γ-quantile of N(0,1).

- $\beta = 0\%$: no uncertainty (20 independent N(0,1)).
- $\beta = 50\%$: full uncertainty.

$\mathcal{F} = [q_\beta, q_{1-\beta}]^d$	$\mathcal{U} = \emptyset$ $\beta = 0\%$			$\mathcal{U} = \mathbb{R}^d$ $\beta = 50\%$
$\rho = 0$	4.47			(0 , 20)

Example $d = 20$ risks N(0,1)

- $(X_1, ..., X_{20})$ independent N(0,1) on

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- $\beta = 0\%$: no uncertainty (20 independent N(0,1))
- $\beta = 50\%$: full uncertainty

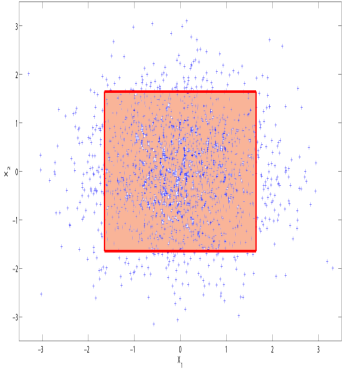
$\mathcal{F} = [q_\beta, q_{1-\beta}]^d$	$\mathcal{U} = \emptyset$ $\beta = 0\%$	$p_f \approx 98\%$ $\beta = 0.05\%$	$p_f \approx 82\%$ $\beta = 0.5\%$	$\mathcal{U} = \mathbb{R}^d$ $\beta = 50\%$
$\rho = 0$	4.47	(4.4 , 5.65)	(3.89 , 10.6)	(0 , 20)

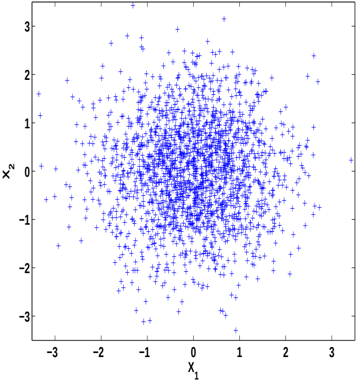
Model risk on the volatility of a portfolio is reduced a lot by incorporating information on dependence!

Information on the joint distribution

- Can come from a fitted model
- Can come from experts' opinions
- Dependence "known" on specific scenarios

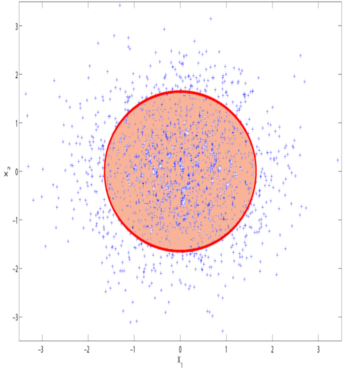
Illustration with marginals N(0,1)

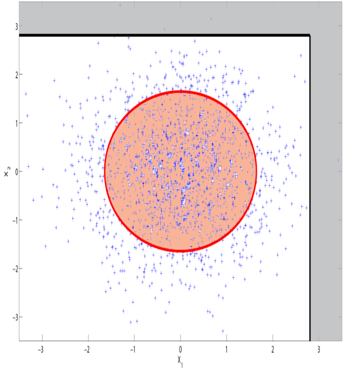




$$\mathcal{F}_1 = \bigcap_{k=1}^{2} \{q_\beta \leq X_k \leq q_{1-\beta}\}$$

Illustration with marginals N(0,1)





$\mathcal{F}_1$ =contour of MVN at β

$$\mathcal{F} = \bigcup_{k=1}^{2} \{X_k > q_p\} \bigcup \mathcal{F}_1$$

Part II

Bounds on Value-at-Risk

VaR aggregation with dependence uncertainty
Our findings

- Maximum Value-at-Risk is not caused by the comonotonic scenario.
- Maximum Value-at-Risk is achieved when the variance is *minimum* in the tail. The RA is then used in the tails only.
- Bounds on Value-at-Risk at high confidence level stay wide even when the trusted area covers 98% of the space!

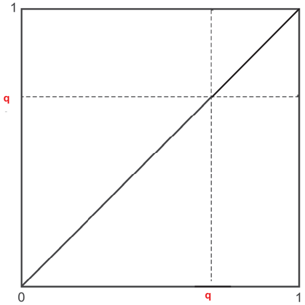
Risk Aggregation and full dependence uncertainty
Literature review

- Marginals known
- Dependence fully unknown (too wide bounds, all info. ignored)
- Explicit sharp (attainable) bounds
  - $n = 2$ (Makarov (1981), Rüschendorf (1982))
  - Rüschendorf & Uckelmann (1991), Denuit, Genest & Marceau (1999), Embrechts & Puccetti (2006),
- A challenging problem in $n \geq 3$ dimensions
- Approximate sharp bounds
  - Puccetti and Rüschendorf (2012): algorithm (RA) useful to approximate the minimum variance.
  - Embrechts, Puccetti, Rüschendorf (2013): algorithm (RA) to find bounds on VaR

"Riskiest" Dependence: maximum VaR_q in 2 dims?

If X_1 and X_2 are U(0,1) comonotonic, then

$$VaR_q(S^c) = VaR_q(X_1) + VaR_q(X_2) = 2q.$$



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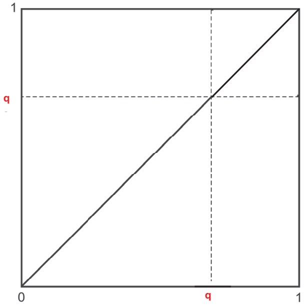
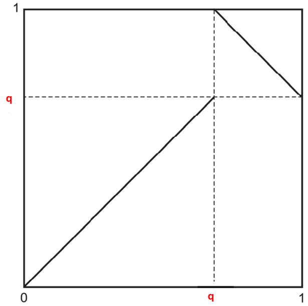


Note that $TVaR_q(S^c) = \frac{\int_q^1 2pdp}{1-q} = 1 + q$ (which is also MAX TVaR)

"Riskiest" Dependence: maximum VaR_q in 2 dims

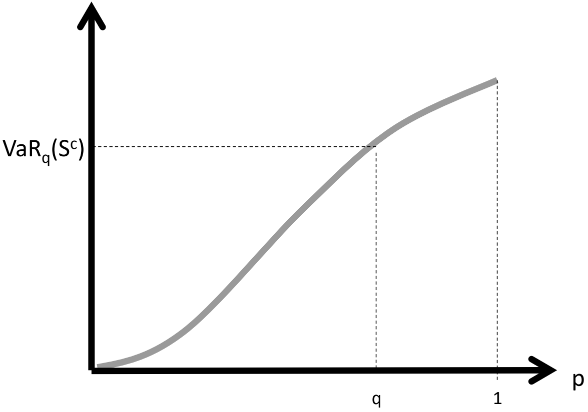
If X_1 and X_2 are U(0,1) and antimonotonic in the tail, then $VaR_q(S^*) = 1 + q$ (which is maximum possible).



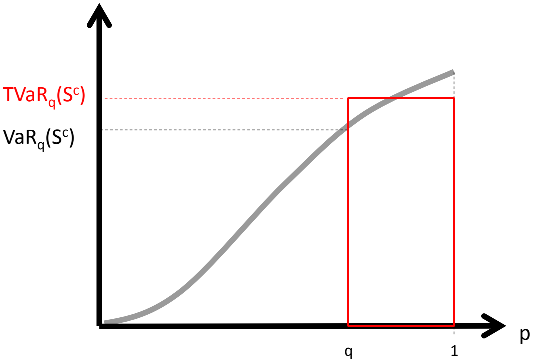
$$VaR_q(S^*) = 1 + q > VaR_q(S^c) = 2q$$

$\Rightarrow$ to maximize VaR_q, the idea is to change the comonotonic dependence such that the sum is constant in the tail

VaR at level q of the comonotonic sum w.r.t. q



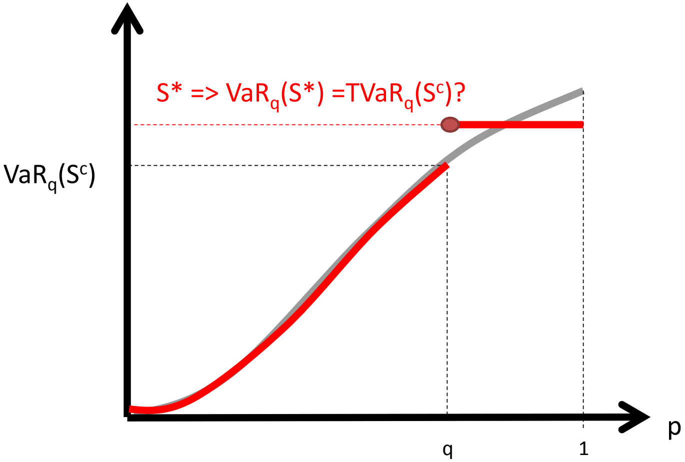
VaR at level q of the comonotonic sum w.r.t. q



$$\text{where TVaR (Expected shortfall):} \mathrm{TVaR}_q(X) = \frac{1}{1-q}\int_q^1 \mathrm{VaR}_u(X)\mathrm{d}u,$$

$$q \in (0,1)$$

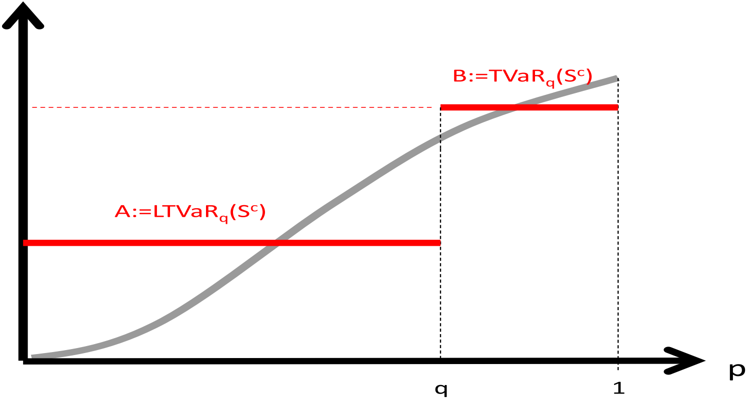
Riskiest Dependence Structure VaR at level q



Analytic expressions (not sharp)

Analytical Unconstrained Bounds with $X_j \sim F_j$

$$A = LTVaR_q(S^c) \leq \mathrm{VaR}_q\left[X_1 + X_2 + ... + X_n\right] \leq B = TVaR_q(S^c)$$



Approximate sharp bounds:

Embrechts, Puccetti, Rüschendorf (2013): algorithm (RA) to find bounds

Numerical Results for VaR, 20 risks $N(0,1)$

When marginal distributions are given,

- What is the maximum Value-at-Risk?
- What is the minimum Value-at-Risk?
- A portfolio of 20 risks normally distributed N(0,1). Bounds on VaR_q (by the rearrangement algorithm applied on each tail)

q=95%	(-2.17 , 41.3)
q=99.95%	(-0.035 , 71.1)

- Very wide bounds
- All dependence information ignored

Idea: add information on dependence from a fitted model or from experts' opinions

Information on a subset

VaR bounds when the joint distribution of $(X_1, X_2, ..., X_n)$ is known on a subset of the sample space.

Our assumptions on the cdf of $(X_1, X_2, ..., X_n)$

$\mathcal{F} \subset \mathbb{R}^n$ ("trusted" or "fixed" area)
$\mathcal{U} = \mathbb{R}^n \backslash \mathcal{F}$ ("untrusted").
We assume that we know:

(i) the marginal distribution F_i of X_i on $\mathbb{R}$ for $i = 1, 2, ..., n$,
(ii) the distribution of $(X_1, X_2, ..., X_n) \mid \{(X_1, X_2, ..., X_n) \in \mathcal{F}\}$.
(iii) $P((X_1, X_2, ..., X_n) \in \mathcal{F})$

- **Goal: Find bounds on $\text{VaR}_q(S) := \text{VaR}_q(X_1 + ... + X_n)$ when $(X_1, ..., X_n)$ satisfy (i), (ii) and (iii).**

Numerical Results, 20 correlated $N(0,1)$ on $\mathcal{F} = [q_\beta, q_{1-\beta}]^n$

$\mathcal{F}$	$\mathcal{U} = \emptyset$ $\beta = 0\%$			$\mathcal{U} = \mathbb{R}^n$ $\beta = 50\%$
q=95%	12.5			(-2.17 , 41.3)
q=99.5%	19.6			(-0.29 , 57.8)
q=99.95%	25.1			(-0.035 , 71.1)

- $\mathcal{U} = \emptyset$: 20 correlated standard normal variables ($\rho = 0.1$).

$$\text{VaR}_{95\%} = 12.5 \quad \text{VaR}_{99.5\%} = 19.6 \quad \text{VaR}_{99.95\%} = 25.1$$

Numerical Results, 20 correlated $N(0,1)$ on $\mathcal{F} = [q_\beta, q_{1-\beta}]^n$

	$\mathcal{U} = \emptyset$ $\beta = 0\%$	$p_f \approx 98\%$ $\beta = 0.05\%$	$p_f \approx 82\%$ $\beta = 0.5\%$	$\mathcal{U} = \mathbb{R}^n$ $\beta = 50\%$
q=95%	12.5	(12.2 , 13.3)	(10.7 , 27.7)	(-2.17 , 41.3)
q=99.5%	19.6	(19.1 , **31.4**)	(16.9 , **57.8**)	(-0.29 , **57.8**)
q=99.95%	25.1	(24.2 , **71.1**)	(21.5 , **71.1**)	(-0.035 , **71.1**)

- $\mathcal{U} = \emptyset$: 20 correlated standard normal variables ($\rho = 0.1$).

$$\text{VaR}_{95\%} = 12.5 \quad \text{VaR}_{99.5\%} = 19.6 \quad \text{VaR}_{99.95\%} = 25.1$$

- **The risk for an underestimation of VaR is increasing in the probability level used to assess the VaR.**
- **For VaR at high probability levels ($q = 99.95\%$), despite all the added information on dependence, the bounds are still wide!**

Regulation challenge

The Basel Committee (2013) insists that **a desired objective of a Solvency framework concerns comparability**:

"*Two banks with portfolios having identical risk profiles apply the frameworks rules and arrive at the same amount of risk-weighted assets, and two banks with different risk profiles should produce risk numbers that are different proportionally to the differences in risk*"

How does correlation impact Value-at-Risk bounds?

Carole Bernard [1, 2] Corrado De Vecchi [1] Steven Vanduffel [1]

[1] Vrije Universiteit Brussel

[2] Grenoble Ecole de Management

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Upper bound for $\text{VaR}_q^+(X_1 + X_2)$

Assume X_i has marginal cdf F_i and C denotes the copula for (X_1, X_2)

$\delta(C, F_1, F_2)$= Spearman's rho, Kendall's tau or Pearson correlation.

$$\begin{aligned} \overline{\text{VaR}}_q^d := \sup \quad & \text{VaR}_q^+(X_1 + X_2) \\ \text{subject to} \quad & X_j \sim F_j, \ j = 1, 2 \\ & \delta(C, F_1, F_2) = d. \end{aligned} \tag{1}$$

Unconstrained problem:

$$\begin{aligned} \overline{\text{VaR}}_q := \sup \quad & \text{VaR}_q^+(X_1 + X_2) \\ \text{subject to} \quad & X_j \sim F_j, \ j = 1, 2. \end{aligned} \tag{2}$$

Upper bound for $\mathrm{VaR}_q^+(X_1 + X_2)$: copulas

Given $q \in (0,1)$, consider the squares $[0,q]^2$ and $[q,1]^2$.

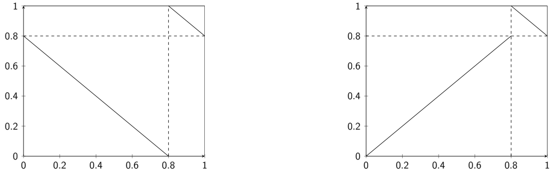


Figure: Supports of C_{min} (left) and C_{max} (right) for $q = 0.8$.

Definition:

Given $q \in (0,1)$, let δ be a measure of dependence (Kendall's tau, Spearman's rho or Pearson correlation), F_1 and F_2 two c.d.f, we define

$$\delta_{min} = \delta(C_{min}, F_1, F_2)$$

$$\delta_{max} = \delta(C_{max}, F_1, F_2)$$

Upper bound for $\text{VaR}_q^+(X_1+X_2)$: results

$$\overline{\text{VaR}}_q^d := \sup \quad \text{VaR}_q^+(X_1+X_2)$$
$$\text{subject to} \quad X_j \sim F_j,\ j=1,2 \qquad (3)$$
$$\delta(C, F_1, F_2) = d.$$

Theorem:

Given $q \in (0,1)$, let δ be a measure of dependence (Kendall's tau, Spearman's rho or Pearson correlation), F_1 and F_2 two c.d.f.
For every $d \in [\delta_{min}, \delta_{max}]$ it holds that

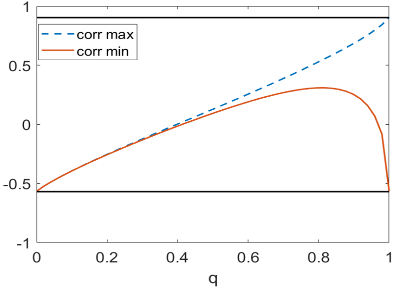
$$\overline{\text{VaR}}_q^d = \overline{\text{VaR}}_q. \qquad (4)$$

and the upper bound is attained.

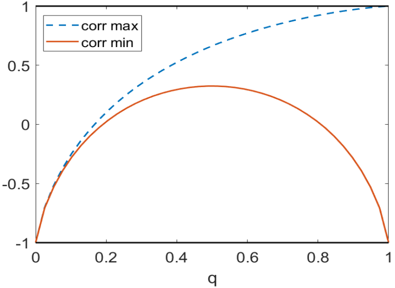
Note $d \in [\delta_{min}, \delta_{max}] \implies$ **constraint is redundant**

Upper bound for $\text{VaR}_q^+(X_1 + X_2)$: results

- Fix δ and d. If $d \in [\delta_{min}, \delta_{max}]$, then $M(d) = \overline{M}$.
- $[\delta_{min}, \delta_{max}]$ is easy to compute.
- for $q \geq 0.95$, $[\delta_{min}, \delta_{max}]$ almost covers the range of values for δ.



A: $X_1 \sim Gamma(2, 3)$,
$X_2 \sim Lognormal(2, 1)$,



B: $X_i \sim N(0, 1), i = 1, 2$.

Interval $[\delta_{min}, \delta_{max}]$

1. δ_{min} and δ_{max} are very easy to compute:
   - Spearman's rho:

$$\rho_{min} = -6q\,(q-1) - 1 \quad \text{and} \quad \rho_{max} = 1 - 2(1-q)^3.$$

   - Kendall's tau:

$$\tau_{min} = -4q\,(q-1) - 1 \quad \text{and} \quad \tau_{max} = -2\,(q-1)^2 + 1.$$

2. for $q \approx 1$, $[\delta_{min}, \delta_{max}]$ almost covers the range of values of δ.

Table: δ=Spearman's rho, range $[-1, 1]$.

q	δ_{min}	δ_{max}
95.0%	-0.715	0.999
99.0%	-0.941	0.999
99.5%	-0.970	0.999

RVaR bounds with n risks

Average correlation: given a portfolio $\mathbf{X} = (X_1, ..., X_n)$, the average correlation of $\mathbf{X}$, $\text{acorr}(\mathbf{X})$, is defined as

$$\text{acorr}(\mathbf{X}) = \frac{\sum_{i \neq j} corr(X_i, X_j) std(X_i) std(X_j)}{\sum_{i \neq j}^{n} std(X_i) std(X_j)}. \tag{5}$$

Range Value-at-Risk:

$$\text{RVaR}_{\alpha,\beta}(X) = \frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} \text{VaR}_{\gamma}(X) d\gamma, \ 0 < \alpha < \beta < 1. \tag{6}$$

Problems:

$$\sup \setminus \inf \left\{ \text{RVaR}_{\alpha,\beta}(S) \, \middle| \, S = \sum_{i=1}^{n} X_i, \ X_i \sim F_i \right\}. \tag{7}$$

$$\sup \setminus \inf \left\{ \text{RVaR}_{\alpha,\beta}(S) \, \middle| \, S = \sum_{i=1}^{n} X_i, \ X_i \sim F_i, \ \text{acorr}(\mathbf{X}) \leq d \right\}. \tag{8}$$

RVaR bounds

1. No dependence information: given $X_i \sim F_i$,

$$A(\beta) \leq \text{RVaR}_{\alpha,\beta}(S) \leq B(\alpha). \quad (9)$$

2. Average correlation constraint: given $X_i \sim F_i$ and $\text{acorr}(\mathbf{X}) \leq d$,

$$l(\beta) \leq \text{RVaR}_{\alpha,\beta}(S) \leq u(\alpha). \quad (10)$$

- Sharpness: tail mixability (sufficient).
- VaR and TVaR bounds as special cases.
- If $d \geq \max(c(\alpha), c(\beta))$, then $l(\beta) = A(\beta)$ and $u(\alpha) = B(\alpha) \implies$ **constraint is redundant**.

Conclusions

Pitfall to avoid:

- Knowledge of dependence measure (such as a correlation coefficient or an average correlation) may not help to improve a risk measure worst-case scenario.

With Value-at-Risk, only tail information helps. In the paper, we also show that

- Knowledge (or realistic assumption) regarding tail dependence is more effective.

Thank you for listening !